Breaking the Heterophily Mixing Barrier in Graph Learning: A Mixed-Curvature Product Manifold Approach

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Introduction

Challenge — GNNs Fail under Heterophily

- Due to the message passing mechanism, GNNs have achieved remarkable success on homophilic graphs, where connected nodes usually share the same label.
- But the performance drops sharply on **heterophilic graphs**, where **connected nodes often belong to different classes**.

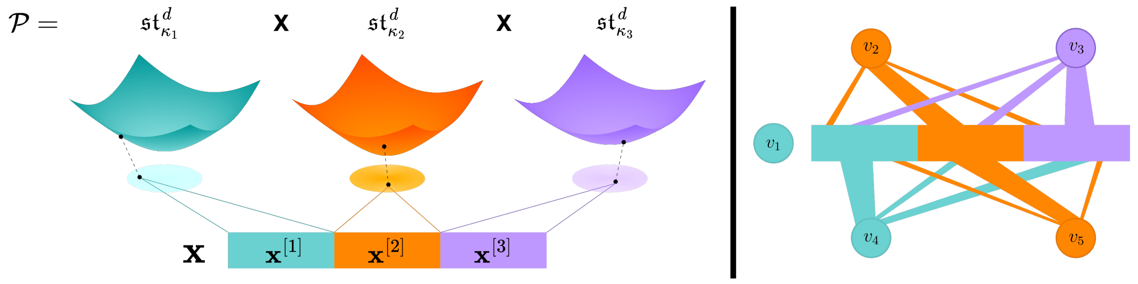
Core Issue — Heterophily Mixing

- Messages from dissimilar classes **become entangled during aggregation**, diluting class-discriminative information.

Our Solution — XMan-GNN Routes Messages by Class

- We assume that **nodes of different classes lie on distinct geometric manifolds**, and thus model nodes on a **mixed-curvature product manifold**.
- During message passing, **information is constrained to flow within class-specific submanifolds**, preventing semantic interference from heterogeneous neighbors.

Methodology



Step 1: Attention-based Soft Label Assignment

$$\mathbf{h}_i^{(l)} = \mathbf{W}_{proj}^{(l)} \mathrm{Log}_{\mathbf{0}}^{\mathcal{K}}(\mathbf{x}_i^{(l-1)}) \qquad \mathbf{s}_{i,j}^{(l)} = \mathrm{Softmax}\left(\frac{\mathbf{W}_{att}^{(l)} \sigma(\mathbf{h}_i^{(l)} + \mathbf{h}_j^{(l)})}{\sqrt{C}}\right)$$

Step 2: Manifold-aware Message Aggregation

$$\mathbf{m}_{i,j}^{(l)}[r] = \mathbf{s}_{i,j}^{(l)}[r] \otimes_{\kappa} \exp_{\mathbf{0}}^{\kappa_r}(\mathbf{h}_j^{(l)})$$

$$\mathbf{z}_i^{(l)}[r] = \bigodot_{v_j \in \mathcal{N}(v_i)} [\mathbf{m}_{i,j}^{(l)}[r], 1]_{\kappa_r} = \frac{1}{2} \otimes_{\kappa_r} \left(\frac{\sum_{v_j \in \mathcal{N}(v_i)} \lambda^{\kappa_r}_{\mathbf{m}_{i,j}^{(l)}[r]} \mathbf{m}_{i,j}^{(l)}[r]}{\sum_{v_j \in \mathcal{N}(v_i)} (\lambda^{\kappa_r}_{\mathbf{m}_{i,j}^{(l)}[r]} - 1)}\right)$$

Step 3: Initial Residual Connections

$$\mathbf{x}_i^{(l)} = \frac{1}{2} \otimes_{\mathcal{K}} \left(\frac{\alpha \lambda^{\mathcal{K}}_{\mathbf{z}_i^{(l)}} \mathbf{z}_i^{(l)} + (1-\alpha) \lambda^{\mathcal{K}}_{\mathbf{x}_i^{(0)}} \mathbf{x}_i^{(0)}}{\alpha \lambda^{\mathcal{K}}_{\mathbf{z}_i^{(l)}} + (1-\alpha) \lambda^{\mathcal{K}}_{\mathbf{x}_i^{(0)}} - 1}\right)$$

Note: All are Hyperbolic Operations

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➢ Performance on *heterophilic* graphs

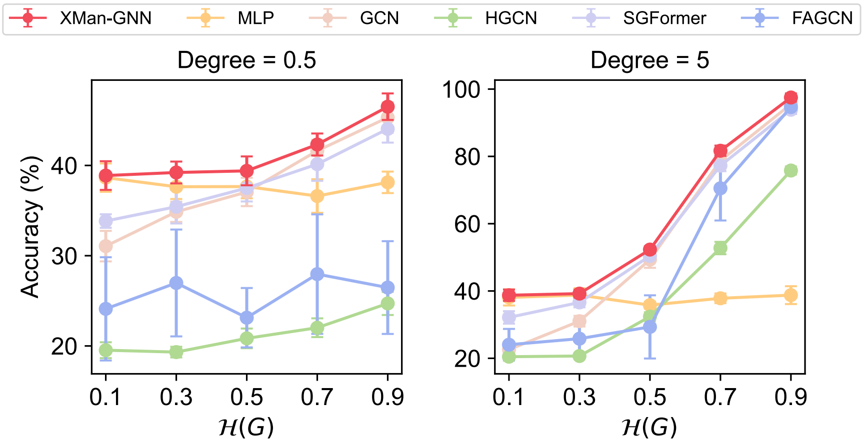
	Texas	Wisconsin	Cornell	Actor	Squirrel	Chameleon	Amazon-Ratings	BlogCatalog	Avg. Rank
MLP	78.65±4.75	84.31±3.28	74.86±4.84	34.64±0.98	34.11±1.80	52.08±2.13	46.73±0.85	93.07±0.30	9.8
GCN	48.11±6.82	48.82±5.51	38.92±7.27	27.32±0.98	27.12±1.13	40.46±2.27	47.97±0.64	77.81±0.89	18.4
GAT	46.22±7.78	49.41±5.02	45.95±5.92	28.38±1.32	29.93±1.36	44.28±2.19	46.74±0.92	63.72±5.15	17.9
HGCN	64.59±10.22	64.31±5.17	52.16±4.84	24.71±0.70	57.53±1.91	67.36±1.14	44.35±0.59	79.34±1.46	13.1
HyboNet	61.08±6.96	59.41±4.48	49.73±8.82	25.29±0.96	53.49±1.93	70.39±2.32	43.25±0.25	74.20±1.02	14.5
HypFormer	80.27±4.69	84.12±4.59	72.16±6.29	33.40±0.82	33.57±1.93	47.37±1.90	**51.93±0.60**	94.59±0.93	9.6
SGFormer	60.54±4.22	63.33±7.34	52.16±4.99	33.05±1.74	32.19±2.58	41.58±2.50	42.99±0.50	96.57±0.41	14.6
GOAT	54.32±6.33	56.08±5.05	46.22±6.45	30.00±3.35	33.85±1.21	38.73±7.23	50.28±2.15	92.26±0.87	16.0
PolyFormer	60.54±6.30	60.20±6.39	57.30±5.77	32.69±2.63	42.96±1.65	58.36±1.70	40.84±3.69	92.15±1.68	14.4
Polynormer	76.76±8.73	77.65±3.64	71.08±3.43	35.29±1.44	38.14±1.27	50.33±1.15	43.99±0.44	96.59±0.42	9.5
H2GCN	80.81±5.05	76.47±5.26	69.19±5.57	32.99±1.03	33.29±1.64	56.07±2.05	43.17±0.50	96.03±0.67	11.1
GPR-GNN	74.59±4.55	77.65±4.57	71.62±7.18	34.85±0.78	31.60±0.84	40.70±1.74	43.44±0.27	95.24±0.44	12.4
FAGCN	67.30±4.90	64.51±3.77	60.27±4.84	35.89±0.94	35.75±1.54	50.88±1.80	42.21±0.41	93.72±0.91	12.4
GloGNN	75.95±5.60	83.92±5.60	72.97±5.41	35.45±1.17	57.54±1.39	71.32±[illegible]	44.6[illegible]	92.17±0.56	8.4
ACM-GCN	87.84±4.40	88.43±3.22	80.34±5.89	35.82±1.09	54.40±1.88	66.93±1.85	38.52±0.43	94.98±0.68	6.9
GOAL	83.62±6.72	86.98±4.46	80.68±6.29	35.96±1.04	60.53±1.65	71.65±1.68	31.94±[illegible]	95.12±0.53	6.1
AERO-GNN	75.95±5.98	68.04±12.34	65.95±3.67	35.74±1.59	36.00±1.15	52.59±2.24	37.47±0.27	95.78±0.73	11.4
Ordered GNN	83.78±5.27	86.47±4.42	75.68±3.20	36.09±0.81	37.32±1.27	50.61±2.09	51.02±0.82	95.27±0.38	6.0
PCNet	60.81±7.28	81.37±3.19	70.27±4.83	35.70±0.84	30.63±1.93	43.73±4.52	38.11±0.74	93.82±1.01	13.9
M2M-GNN	86.22±4.09	88.82±4.21	77.57±3.64	35.91±1.15	**63.20±1.71**	75.37±1.68	50.19±0.69	**96.92±0.48**	2.8
XMan-GNN	**90.27±3.46**	**90.59±3.31**	**84.86±2.16**	**37.07±0.58**	62.46±1.39	**76.63±0.76**	50.89±0.65	96.89±0.37	**1.5**

Avg. Rank: 1.5

➢ Performance on *homophilic* graphs

- XMan-GNN also achieves the best results across all homophilic datasets.

➢ Evaluation on synthetic dataset



➢ Over-smoothing analysis